Estimating the replicability of science by taking statistical significance into account

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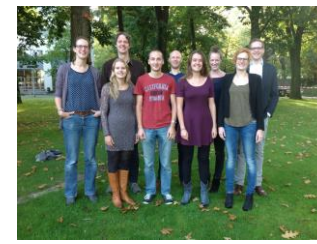

The Problem

Example (Maxwell et al., 2015)

Independent sample t-test

Original: $d = 0.5$, $t(78) = 2.24$, $p = 0.028$

Replication (power = .8 at $d = 0.5$): $d = 0.23$, $t(170) = 1.50$, $p = 0.135$

Conclusion?!?

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Questions considered relevant

1) Does effect exist? (0 or not)
2) What is magnitude of effect? (best guess)

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Conclusion?!?

Questions considered relevant

1) Does effect exist? (0 or not)
   A) No
   B) Yes

The Problem

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Conclusion?!?

Questions considered relevant

2) What is magnitude of effect? (best guess)
   A) 0
   B) (0, 0.23]
   C) (0.23, 0.5]

Omnipresent and Relevant

- Reproducibility Project Psychology (RPP):
  - Significant original study and non-significant replication in 63.9%
- Experimental Economics Replication Project (EE-RP):
  - Significant original study and non-significant replication in 31.2%
- Replication is often a starting point of a multi-study paper

Problem and Solution

Problem

How to evaluate results of original study and replication?

Solution

Accurate evaluation of effect size …

… taking statistical significance of the original study into account

The Message

(1) Methods *should* take statistical significance of original study into account

(2) We developed such a method within a Bayesian framework

(3) Need huge sample sizes (n_i~1,000) to distinguish 0 from small effect

→*With current sample sizes in psychology, one or two studies is not sufficient to accurately evaluate effect size*

(4) Application of method to RPP and EE-RP:

→ *Often not sufficient information for determining magnitude of effect size*

→ *Studied effects larger in EE-RP than RPP*

Overview

1. Publication bias
2. Why we should take significance of original study into account
3. Snapshot Bayesian Hybrid Meta-Analysis Method
4. Statistical properties of snapshot method
5. Application: RPP and EE-RP
6. Compute required sample size with snapshot method
7. Conclusion and discussion

1. Publication bias

- Publication bias is 'the selective publication of studies with a statistically significant outcome'
- Overwhelming evidence of publication bias:
  - 95% of published articles contain significant results in psychology
- Consequences of publication bias:
  - False impression that effect exists
  - Overestimation of effect sizes
  - Questionable research practices

2. Why we should take significance of original study into account

Assume researcher's goal: replicate significant original

i. Selection of high score

ii. Score subject to (sampling) error

→ *Regression to the mean*: expected value of replication is smaller than of original study

! Holds irrespective of publication bias !

Assume researcher's goal: replicate original regardless of significance

No researcher's selection of high score, but…

Selection of high score *through publication bias* → regression to the mean still holds, and should still take significance original study into account

3. Snapshot method: Basic idea

- **Snapshot** Bayesian Hybrid Meta-Analysis Method
  - Assume four effect sizes (zero, small, medium, large [Cohen]) → *snapshots*
- Snapshot **Bayesian** Hybrid Meta-Analysis Method
  - Compute posterior probability of these four effects → *Bayesian*
- Snapshot Bayesian **Hybrid** Meta-Analysis Method
  - Take statistical significance of original study into account → *hybrid*
- Snapshot Bayesian Hybrid **Meta-Analysis** Method
  - Combine original study with replication → *meta-analysis*

3. Snapshot method: Basic idea

- Density of the replication is "normal" pdf because no selection:

$$f_R = f(y = y_R; \theta)$$

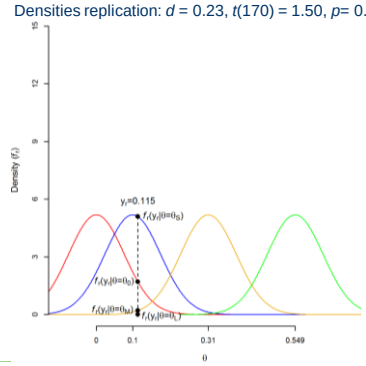
- Density of the original study is pdf conditional on effect size being statistically significant:

$$f_O = \frac{f(y = y_O; \theta)}{P(y \geq y_{CV}; \theta)}$$

- Assumptions:
  - Original study is statistically significant
  - Both studies estimate the same effect (fixed-effect)
  - No questionable research practices

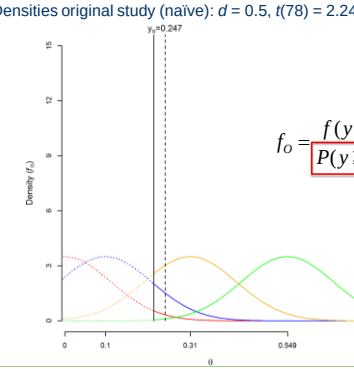
3. Snapshot method: Basic idea

Densities replication: d = 0.23, $t(170)$ = 1.50, p= 0.135



3. Snapshot method: Basic idea

Densities original study (naïve): d = 0.5, $t(78)$ = 2.24, p = 0.028



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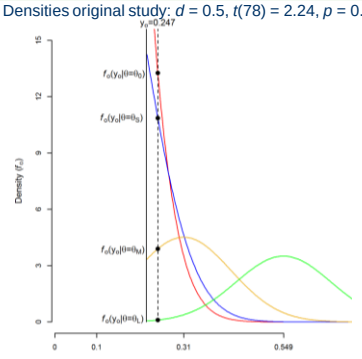

3. Snapshot method: Basic idea

- Combined likelihood:

$$L(\theta) = f_O(\theta) \times f_R(\theta)$$

- Posterior probabilities assuming a uniform prior for each snapshot are computed with:

$$\pi_x = \frac{L(\theta = x)}{L(\theta = \theta_0) + L(\theta = \theta_S) + L(\theta = \theta_M) + L(\theta = \theta_L)}$$

Advantages of method

- Easy and insightful
- Easy (re)computation posterior for other (than uniform) prior:

$$\pi_x^* = \frac{p_x \pi_x}{p_0 \pi_0 + p_S \pi_S + p_M \pi_M + p_L \pi_L}$$

3. Snapshot method

Applied to example Maxwell et al. (2015):

Original: $d = 0.5$, $t(78) = 2.24$, $p = 0.028$

Replication (power = .8 at $d = 0.5$): $d = 0.23$, $t(170) = 1.50$, $p = 0.135$

Hypothesis	Zero	Small	Medium	Large
Naïve	0.063	0.866	0.071	0.000
Snapshot				

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Hypothesis	Zero	Small	Medium	Large
Naïve	0.063	0.866	0.071	0.000
Snapshot	0.287	0.703	0.010	0.000

Evidence of zero effect increased; best guess = small effect

3. Snapshot method

Applied to example Maxwell et al. (2015):

- Other than uniform prior: two times higher prior probability to zero effect than any of other effects (p_0=2; p_S=1; p_M=1; p_L=1)

$$\pi_x^* = \frac{p_x \pi_x}{p_0\pi_0 + p_S\pi_S + p_M\pi_M + p_L\pi_L}$$

Hypothesis	Zero	Small	Medium	Large
Naïve	0.063	0.866	0.071	0.000
Snapshot	0.287	0.703	0.010	0.000
p_0=2	0.446	0.546	0.008	0.000

4. Statistical Properties Snapshot Method

- Analytically approximated properties using numerical integration
- Effect size measure: Correlation coefficient
- 5,000 equally spaced cumulative probabilities given significance for original study (α=.025)
- 5,000 equally spaced cumulative probabilities for replication
- Converting probabilities to effect sizes: 5,000 x 5,000 = 25.000.000

4. Statistical Properties Snapshot Method

- Conditions:
  - ρ = 0; 0.1; 0.3; 0.5
  - Sample size (n_i): 31; 55; 96; 300; 1,000
  - Snapshots (ρ_S) = 0; 0.1; 0.3; 0.5
  - Snapshot and naïve method
- Outcome variables:
  - Expected value of posterior probability
  - Probability of strong evidence (π_x > .75 or Bayes Factor > 3)

4. Statistical Properties Snapshot Method

- Expected values of posterior probabilities:

		Snapshot method			
	n_i	ρ_S=0	ρ_S=0.1	ρ_S=0.3	ρ_S=0.5
ρ=0	31	**0.466**	0.36	0.151	0.023
	55	**0.535**	0.375	0.089	0.002
	96	**0.601**	0.368	0.03	0
	300	**0.757**	0.243	0	0
	1,000	**0.948**	0.052	0	0

- Huge sample sizes (n_i~1,000) are required to distinguish 0 from small effect

4. Statistical Properties Snapshot Method

- Expected values of posterior probabilities (WRONG METHOD):

		Snapshot Naïve method			
	n_i	ρ_S=0	ρ_S=0.1	ρ_S=0.3	ρ_S=0.5
ρ=0	31	**0.177**	0.336	0.411	0.076
	55	**0.212**	0.479	0.304	0.005
	96	**0.241**	0.648	0.112	0
	300	**0.338**	0.662	0	0
	1,000	**0.758**	0.242	0	0

- No correction for statistical significance → overestimation

4. Statistical Properties Snapshot Method

- Expected values of posterior probabilities:

	Snapshot method			
n_i	ρ=0	ρ=0.1	ρ=0.3	ρ=0.5
31	0.466	0.351	0.367	0.669
55	0.535	0.403	0.523	0.808
96	0.601	0.481	0.738	0.918
300	0.757	0.745	0.985	0.997
1,000	0.948	0.948	1	1

- Easier to distinguish medium and large effect

4. Statistical Properties Snapshot Method

- Probability of strong evidence (π_x > .75):

	Snapshot method			
n_i	ρ=0	ρ=0.1	ρ=0.3	ρ=0.5
31	0.04	0	0	0.498
55	0.142	0	0.115	0.732
96	0.291	0	0.645	0.895
300	0.641	0.625	0.982	0.997
1,000	0.935	0.933	1	1

- Large sample size needed for zero and small effect

4. Statistical Properties Snapshot Method

Conclusions:

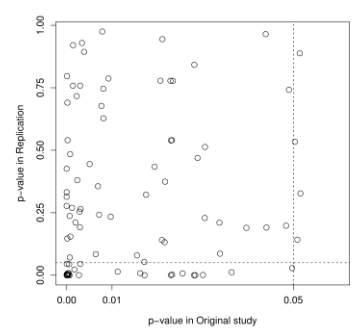
- Not correcting for statistical significance (naïve method) is inappropriate
- Huge sample sizes are required to distinguish 0 from small effect
- Large sample sizes are required for medium and large effect

5. Application: RPP and EE-RP

- Initiatives to study the replicability of psychological and economic research
- RPP: Studies from JPSP, Psychological Science, and Journal of Experimental Psychology: 67 out of 100 studies were included
- EE-RP: Experimental research from the American Economic Review and Quarterly Journal of Economics: 16 out of 18 studies were included
- "High-powered" replication of a key effect

5. Application: RPP and EE-RP

- Distribution of *p*-values in RPP:



5. Application: RPP and EE-RP

- Probability of strong evidence ($\pi_x > .75$) using snapshot method:

	ρ_S				
	0	**0.1**	**0.3**	**0.5**	**Unknown**
EE-RP	0	0.062	0.312	0.438	0.188
RPP	0.134	0.030	0.045	0.164	0.627

- Conclusions:
  - Studied effects larger in EE-RP than in RPP
  - Only few studies have strong evidence for zero effect in RPP (13.4%)
  - Often not enough information for determining magnitude of effect size in RPP (62.7%)

6. Determining sample size with snapshot

- Computing sample size replication to achieve a certain posterior probability akin to power analysis: $P(\pi_x \geq a)=b$
- Approximate distribution of replication's effect size with numerical integration
- Compute posterior probability for each snapshot at different true effect size
- Compute required sample size with and without information of original study

6. Determining sample size with snapshot

Applied to example of Maxwell et al. (2015):

- Original study: r_o=0.243 and n_i=80 (p=.029)

	With original study	**Without original study**
ρ=0	587	645
ρ=0.1	709	664
ρ=0.3	223	215
ρ=0.5	284	116

7. Conclusion and discussion

(1) Methods *should* take statistical significance of original study into account

(2) We developed such a method within a Bayesian framework

(3) Need huge sample sizes (n_i~1,000) to distinguish 0 from small effect
→*With current sample sizes in psychology, one or two studies is not sufficient to accurately evaluate effect size*

(4) Application of method to RPP and EE-RP:
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7. Conclusion and discussion

- R code for snapshot method in "puniform" package and web application: https://rvanaert.shinyapps.io/snapshot/
- Determining sample size of replication with snapshot method akin to computing required sample size with power analysis
- Intervals of effect sizes instead of discrete values as snapshots
- Future research:
  - Extend method such that it can deal with multiple original studies and replications

Thank you for your attention